



MATHEMATICAL MODELING IN DESIGN

Math and physics are very much part of the design process!



Model

- ❑ A **model** is a simplified representation or abstraction of reality.
- ❑ Reality is generally too complex to copy exactly.
- ❑ Much of the complexity is actually irrelevant in problem solving.

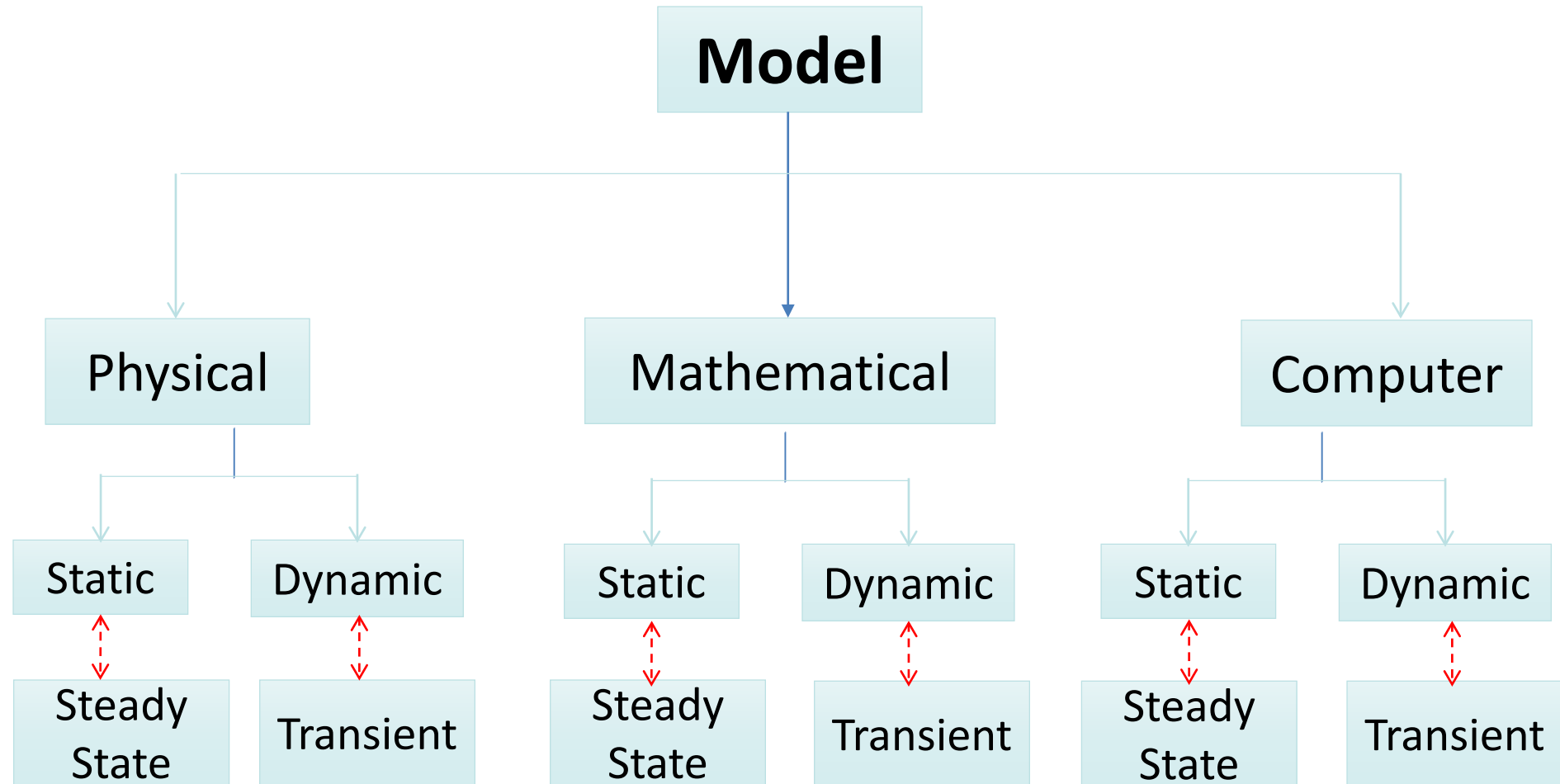


Why do we need modeling?

- ❑ Modeling is required to predict the behavior of a system (or something to be built or fabricated)
- ❑ It is impractical to make a very large number of prototypes and often it is impossible
- ❑ Mathematical modeling is particularly useful when physical models cannot be made or fail to provide enough information
- ❑ It is crucial to understand the performance of a system under extreme (worst) conditions
- ❑ Mathematical modeling enables designers to predict the performance of a design under various conditions



Types of Models





What is Mathematical Model?

- ❑ A set of mathematical equations (e.g., differential eqs.) that describes the input-output behavior of a system
- ❑ Solution to these equations enables the designers to predict the behavior of the physical system in different situations. Often mathematical governing equations with appropriate boundary and initial conditions help solving real world problems

What is a model used for?

- ❑ Simulation
- ❑ Prediction/Forecasting
- ❑ Diagnostics/Prognostics
- ❑ Design/Performance Evaluation
- ❑ Control System Design



Mathematical Modeling Stages

In general there are 6 stages in mathematical modeling

- ❑ Stage 1: Specify the problem & Define the objectives:
The physical system may be divided into subsystems to provide a clear understanding of the problem and its objectives
- ❑ Stage 2: Setting up a model
 - ❑ Making decisions on features which should be included in the model and those that can be neglected. Determine the crucial variables
- ❑ Stage 3: Formulation of the model
 - ❑ Determine the information about the real system which should be translated into equations or mathematical statements (Gov. Eqs & BCs, ICs), which is often the most difficult stage



Mathematical Modeling Stages

- ❑ Stage 4: Solving the mathematical problem
 - ❑ select a method of solution (**analytical or numerical**)
 - ❑ **Considerations:** Geometry, Material Properties, Linearity
 - ❑ Make necessary assumptions to simplify

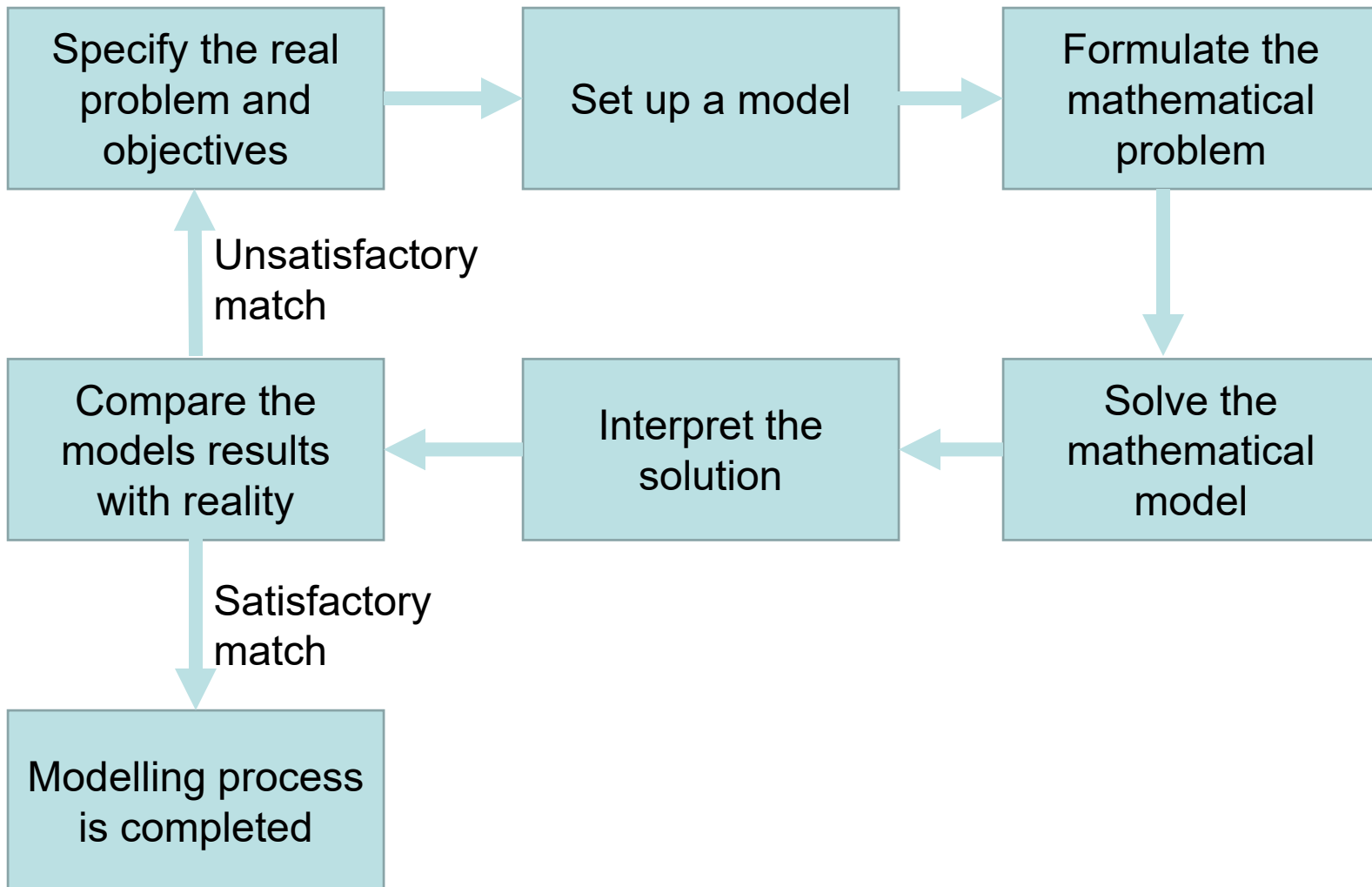
- ❑ Stage 5: Interpretation of the solution
 - ❑ The obtained solution should behave reasonably when changes are made to the parameters
 - ❑ The model behavior for extreme condition should be explainable

- ❑ Stage 6: Compare with reality (Sanity Check)
 - ❑ Validation
 - ❑ Evaluation
 - ❑ Iteration
 - ❑ Engineering Sense

If the comparison is successful, the modeling process is complete. Otherwise, modifications will be necessary throughout the process

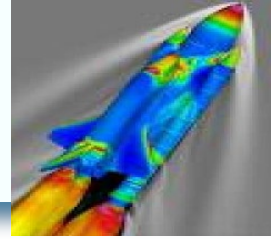


Mathematical Modeling Stages





Mathematical Modeling, Assumptions



- ❑ The most critical part of mathematical modeling is the set of assumptions to be made
- ❑ Assumptions are made to reduce complexity of some physical phenomena in order to simplify the mathematical model and its solution
- ❑ The application of simplifying assumptions is usually associated with loss of accuracy in the prediction, and hence requires experiences and expertise
- ❑ It is important to have a good estimate about the level of inaccuracy due to the applied simplifying assumptions
 - ❑ Example: Air friction may be assumed negligible in predicting the surface temperature of a ball in baseball, however such an assumption on the surface temperature of space shuttle entering the atmosphere will be catastrophic



Mathematical Modeling

- A mathematical model is represented as a functional relationship of the form

Dependent Variable = **f** (Independent Variables, parameters, forcing functions)

- **Dependent/Field variable**: Characteristic that usually reflects the state of the system often the unknown that you are solving the problem for (e.g. **Temp**, **Press**, **δ** , **σ** , **ϵ** , etc.)
- **Independent variables**: Dimensions such as **time** (**t**) and **space** (**x** , **y** , **z**) along which the systems behavior is being determined
- **Parameters**: Reflect the system's properties or composition
- **Forcing functions**: external influences acting upon the system



Three Dimensional Orthotropic Conduction Governing Equation

$$K_x (\partial^2 T / \partial x^2) + K_y (\partial^2 T / \partial y^2) + K_z (\partial^2 T / \partial z^2) + (e_{\text{gen}}) = \rho c (\partial T / \partial t)$$



Newton's 2nd law of Motion

- ❑ Statement: “The time rate change of momentum of a body is equal to the resulting force acting on it.”
- ❑ The model is formulated as

$$\mathbf{F} = \mathbf{m} \mathbf{a}$$

F= Net force acting on the body (N)

m= Mass of the object (kg)

a= Its acceleration (m/s^2)

- ❑ Formulation of this law has several characteristics that are typical of mathematical models of the physical world:
 - ❑ It describes a natural process or system in mathematical terms,
 - ❑ It represents an idealization and simplification of reality,
 - ❑ Finally, it yields reproducible results, consequently, can be used for predictive purposes.



- ❑ Some mathematical models of physical phenomena may be much more complex.
- ❑ Complex models may not be solved exactly or require more sophisticated mathematical techniques than simple algebra for their solution.
 - ❑ Example, modeling of a falling parachutist:





$$\frac{dv}{dt} = \frac{F}{m}$$

$$F = F_D + F_U$$

$$F_D = mg$$

$$F_U = -cv$$

$$\frac{dv}{dt} = \frac{mg - cv}{m}$$

$$\frac{dv}{dt} = g - \frac{c}{m}v$$



- ❑ This is a differential equation and is written in terms of the differential rate of change dv/dt of the variable that we are interested in predicting.
- ❑ If the parachutist is initially at rest ($v=0$ at $t=0$), using calculus

$$v(t) = \frac{gm}{c} \left(1 - e^{-(c/m)t} \right)$$

Diagram illustrating the components of the equation:

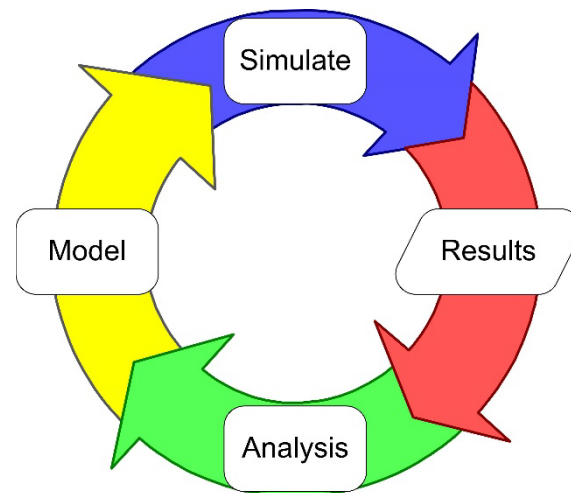
- $v(t)$: Dependent variable
- $\frac{gm}{c}$: Forcing function
- t : Independent variable
- c and m : Parameters



Simulation

- ❑ Computer simulation is the discipline of designing a model of an actual or theoretical physical system, executing the model on a digital computer, and analyzing the execution output.
- ❑ Simulation embodies the principle of “learning by doing” --- to learn about the system we must first build a model of some sort and then operate the model.
- ❑ In an FEA Modeling the following parameters must be modeled accurately to obtain reasonable results:

- ❑ Volume/Geometry
- ❑ Materials Properties
- ❑ ICs & BCs
- ❑ External Loading





Advantages to Simulation

- ☐ Can be used to study existing systems without disrupting the ongoing operations.
- ☐ Proposed systems can be “tested” before committing resources.
- ☐ Allows us to control time.
- ☐ Allows us to identify bottlenecks.
- ☐ Allows us to gain insight into which variables are most important to system performance.