Tension, Compression, Shear

\[ \sigma = \frac{P}{A} \quad (\text{lb/in}^2 = \text{Psi}) \quad (\text{N/m}^2 = \text{Pa}) = \text{Stress} \]

\[ \varepsilon = \frac{\delta L}{L} \quad (\text{in/in}) \quad (\text{m/m}) = \text{Strain} \]

\[ \nu = -\frac{\varepsilon_2}{\varepsilon_1} \quad \text{Poisson’s Ratio} \]

\[ \text{FOS} = \frac{\text{Strength}}{\text{Maximum Stress}} > 1 \]
\[ \sigma = \frac{F}{A} \]

\[ \varepsilon = \frac{\Delta L}{L} \]

\[ \sigma = E \cdot \varepsilon \]

Young's Modulus

Strain

Change in Length

Original Length

Cross-Section Area

Force
\[ \sigma = \frac{MC}{I}, \text{ Cantilevered Beam Bending, } L \]

\[ \sigma = \frac{F (L-x) y}{(bh^3)/12} \quad \text{(lb/in}^2 = \text{psi)} \]
Failure Analysis & Factor of Safety (FOS):

Two Considerations:

(I) Stress Critical Structures
(II) Stiffness (Deflection/Displacement) Critical Structures
(III) Materials Selection Issues Related to Failure Analysis

I. Stress Critical Structures: \( FOS = \frac{\sigma_{\text{yield}}}{\sigma_{\text{max}}} > 1 \)  

VI.1.

I.1. For Isotropic Materials (Metals, Polymers, Ceramics)

Use Maximum Von Mises Stress:

\[ FOS = \left( \frac{\sigma_{\text{yield}}}{\sigma_{\text{max von Mises}}} \right) > 1 \]

I.2. For Anisotropic Materials

(Composite materials)

Use Maximum Stress Criterion: i.e., find FOS for each stress-component (i.e., \( FOS = \text{direction-strength}\text{/maximum direction-stress} \)), and then the Min FOSs is the FOS.

For example:

(1) Find FOSs for Normal Stresses (3 components):

\( FOS | \sigma_{xx} \) : \( FOS | \sigma_{xx} = \left( \frac{\text{Normal Strength in X-Direction}}{\sigma_{xx \text{ max}}} \right) > 1 \)

Likewise for \( \sigma_{yy} \) & \( \sigma_{zz} \) : \( FOS | \sigma_{yy} \) & \( FOS | \sigma_{zz} > 1 \)

(2) Find FOSs for Shear Stresses (3 independent components):

\( FOS | \sigma_{xy} \) : \( FOS | \sigma_{xy} = \left( \frac{\text{Shear Strength in XY-Plane}}{\sigma_{xy \text{ max}}} \right) > 1 \)

Likewise for \( \sigma_{yz} \) & \( \sigma_{zx} \) : \( FOS | \sigma_{yz} \) & \( FOS | \sigma_{zx} > 1 \)

(3) Final FOS = Min (\( FOS | \sigma_{xx} \), \( FOS | \sigma_{yy} \), \( FOS | \sigma_{zz} \), \( FOS | \sigma_{xy} \), \( FOS | \sigma_{yz} \), \( FOS | \sigma_{zx} \)) > 1

Typical FOS in Design = 1.5, Human Life Involved = 2.5 (if confident about your assumption, modeling, and analysis); otherwise pick Larger (up to 4).

In Space Applications since weight is a critical factor, the analyses are conducted accurately, and hence it is afforded to use lower FOSs = \( \sim 1.1 \) to 1.25.
I.3. Remedies if FOS = $\sigma_{\text{yield}} / \sigma_{\text{max}} < 1$:

We want to increase FOS to > 1: So,

(1) Either Increase the Numerator (i.e., $\sigma_{\text{yield}}$)

(2) Or Decrease the Denominator (i.e., $\sigma_{\text{max}}$)

(1) Increase the Numerator (i.e., $\sigma_{\text{yield}}$):

By changing the material to a Stronger Material with higher $\sigma_{\text{yield}}$, e.g., change Aluminum to Steel.

(2) Decrease the Denominator (i.e., $\sigma_{\text{max}}$):

$\sigma$ can, in general, be modeled as $\sigma = F / A$

\[ \sigma_{\downarrow} = F / A_{\uparrow} \]

Since $F$ is a design parameter and often cannot be changed, hence the remedy is to use a “beefier” components, i.e., with higher cross-sectional area, $A$, where stress is acting on.

For example: The beam with cross-section of $b=$width, $h=$thickness, and $l=$length

For Axial Loading (Tension or Compression): $\sigma = F / A_{\uparrow}$, Increase $A = b \times h$

For Lateral Loading (Bending): $\sigma = MC/I$ & $I = b h^3/12$, Increase $h$ since it has cubic effect in increasing $I$ than $b$. 

5
II Stiffness (Deflection/Displacement) Critical Structures:  
Deflection/Displacement < δ_{limit}

II.1. Limits of Deflections/Displacement in Designs:

For those structures that have FOS >> 1, then next consideration is if the material is stiff enough to give Deflection/Displacement < δ_{limit} (assuming the nominal cross-section is 10mmx10mm, i.e., not more than 10% to also stay within Linear Regime)

Therefore, in many designs various components have their own Critical Deflection/Displacement Limits (δ_{limit}) that designers should check with the sponsor/client; however, a rule-of-thumb is that:

Deflection/Displacement < 1 mm=0.040” (for a nominal cross-section: 10mmx10mm to also stay within the Linear Regime)

II.2. Remedies if Deflection/Displacement > δ_{limit}:

The amount of Deflection/Displacement in a structure is a function of the structure Stiffness (i.e., Young’s Modulus, E: σ = E ε).

For Axial Loading (Tension or Compression): σ = F / A = E ε = E ΔL/L:

F = EA ΔL/L =⇒ ΔL = δ = F |L| / E A

Then, if F, L, and A are the design parameters that you cannot change, then the only remedy is to use a Stiffer Materials, e.g., change Aluminum to Steel.

For Lateral Bending Loading (Bending):

\[ f = \delta = \frac{FL^3}{3EI} \], where \( I = bh^3/12 \)

Understanding that L has a cubic effect on δ compared with F; however, they both may be the design parameters that you cannot change. In addition, EI is called “Flexural Rigidity” that should increase in this case. Again, note that to increase I, h has cubic effect in increasing I than b. Finally, to reduce δ, E (i.e., the materials stiffness) should increase, e.g., change Aluminum to Steel.
III. Materials Selection Issues Related to Failure Analysis

If **Weight is a Major Factor in Design**, then:

### III.1. For Stress-Critical Structures:

For Stress-Critical Structures the parameter that should be maximized is not only $\sigma_{\text{yield}}$ (i.e., *Yield Strength*), but $\sigma_{\text{yield}} / \rho$ (i.e., *Specific Strength*), where $\rho$ is the density of the material, **should be maximized**.

**For example**: *(\sigma_{\text{yield}} / \rho)* unit in the following is: “KN*m/Kg”

\[
(\sigma_{\text{yield}} / \rho)_{\text{Composites}} = \frac{C}{E} \approx 2,000^* \quad >> \quad (\sigma_{\text{yield}} / \rho)_{\text{Steel}} = \frac{\sigma_{\text{yield}}}{\rho} \approx 200^* \quad >> \quad (\sigma_{\text{yield}} / \rho)_{\text{Al}} = \frac{\sigma_{\text{yield}}}{\rho} \approx 100^*
\]

### III.2. For Stiffness-Critical Structures:

For Stiffness-Critical Structures the parameter that should be maximized is not only $E$ (i.e., *Young’s Modulus or Stiffness*), but $E / \rho$ (i.e., *Specific Stiffness*), where $\rho$ is the density of the material, **should be maximized**.

**For example**: *(E / \rho)* unit is in “m$^2$ s$^{-2}$ * 10$^6$”.

\[
(E / \rho)_{\text{Composites}} = \frac{C}{E} \approx 100 \quad >> \quad (E / \rho)_{\text{Steel}} = \frac{E}{\rho} \approx 25 \quad = \quad (E / \rho)_{\text{Al}} = \frac{E}{\rho} \approx 25
\]