Finite Element Analysis

Senior Design
ME481
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We perform analysis for:

• Deformations and internal forces/stresses
• Temperatures and heat transfer in solids
• Fluid flows (with or without heat transfer)
• Conjugate heat transfer (between solids and fluids)
• etc...
Effective design

• Performs the required task efficiently
• is inexpensive in materials used
• is safe under extreme operating conditions
• can be manufactured inexpensively
• is pleasing/attractive to the eye
• etc...
Analysis means probing into, modeling, simulating nature

Therefore, analysis gives us insight into the world we live in, and this

Enriches our life

Many great philosophers were analysts and engineers …
Analysis is performed based upon the laws of mechanics

- Mechanics
  - Solid/structural mechanics (Solid/structural dynamics)
  - Fluid mechanics (Fluid dynamics)
  - Thermo mechanics (Thermo dynamics)
The process of modeling for analysis

Physical problem (given by a “design”)

Change of physical problem

Mechanical model

Improve model

Solution of mechanical model

Refine analysis

Interpretation of results

Design improvement
In engineering practice, analysis is largely performed with the use of finite element computer programs (such as NASTRAN, ANSYS, ADINA, SIMULIA, LSDyna, etc…)

These analysis programs are interfaced with Computer-Aided Design (CAD) programs Catia, SolidWorks, Pro/Engineer, NX, etc.
Modeling

Means taking increasingly more complex models to simulate nature with increasing accuracy

Assumptions:
- spring, rod, truss
- beam, shaft
- 2-D solid
- plate
- shell
- fully three-dimensional
- dynamic effects
- nonlinear effects
- nature
In CAD System

CAD solid model is established

In Analysis System

• Preparation of the mathematical model
• Meshing and Solution
• Presentation of results
Numerical Analyses: Discretization Methods

Nature of Numerical Methods
Consists of a set of numbers from which the distribution of the dependent variable can be constructed

\[ \varphi = a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m \]

- Evaluate at any location \( x \), by substituting the values of \( x \) and the values of \( a \)'s into Equation
- Obtain the values of \( \varphi \) at various locations
- Construct a method that employs the values of \( \varphi \) at a given number of points (called grid points)

- Conservation of mass, the conservation of momentum, and the conservation of energy
- A set of algebraic equations for the unknowns
- Prescribing an algorithm for solving the equations

\[
\left( \frac{\partial^2 u}{\partial x \partial y} \right)_{i,j} = \frac{\left( \frac{\partial u}{\partial y} \right)_{i+1,j} - \left( \frac{\partial u}{\partial y} \right)_{i-1,j}}{2\Delta x} + O(\Delta x)^2
\]

\[
\left( \frac{\partial u}{\partial y} \right)_{i+1,j} = \frac{u_{i+1,j+1} - u_{i+1,j-1}}{2\Delta y} + O(\Delta y)^2
\]

\[
\left( \frac{\partial u}{\partial y} \right)_{i-1,j} = \frac{u_{i-1,j+1} - u_{i-1,j-1}}{2\Delta y} + O(\Delta y)^2
\]
Discretization Concept

By assigning values at the grid points:
- Continuous information contained in the exact solution of the differential equation is replaced by discrete values

- Discretization equations are derived from the differential equation governing $\varphi$
- Some assumptions about how $\varphi$ varies between grid points
- Continuum calculation domain has been discretized
- Governing differential equation has been replaced with simple algebraic equations,
- The eqs can be solved with relative ease
Structure of Discretization Equation

- A discretization equation is an *algebraic relation* connecting the values of φ for a group of grid points
- Derived from the *differential equation*
- With *same physical information* as the differential equation
- As the number of grid points become very large, the solution to the discretization equation is expected to approach the exact solution of the corresponding differential equation
- Wont give the exact same solution
Finite Element Method

- Whole Domain and Finite element discretization
- Material property (moduli of elasticity for composites) and geometry vary with spatial coordinate
- Solutions in each of these subdomains are represented by different functions

- Nodes, elements
- Mesh

- Number of Equations
Approximate Solution

- Mesh is smaller if
- abrupt changes in: geometry,
- material properties and
- external conditions
- (load, temperature etc.)

- Mesh convergence and accuracy

https://www.comsol.com/multiphysics/mesh-refinement
Finite Element Representation


http://www.predictiveengineering.com/consulting/ls-dyna-consulting
A reliable and efficient finite element discretization scheme

- for a well-posed mathematical model

- always give,
  for a reasonable finite element mesh,
  a reasonable solution

- if the mesh is fine enough, an accurate solution can be obtained
Element Selection

We want elements that are reliable for any
- geometry
- boundary conditions
- and meshing used

The displacement method is not reliable for
- plates and shells
- almost incompressible analysis
Some analysis experiences

Tremendous advances have taken place –

• **mixed optimal elements** have greatly increased the efficiency and reliability of analyses

• **sparse direct solvers** and **algebraic multigrid iterative solvers** have lifted the analysis possibilities to completely new levels
In Industry:
Two categories of analyses

• Analysis of problems for which test results are scarce or non-existent
  – large civil engineering structures

• Analysis of problems for which test results can relatively easily be obtained
  – mechanical / electrical engineering structures
Examples of category 1 problems

- Analysis of offshore structures
- Seismic analysis of major bridges
  - only "relatively small" components can be tested

Reliable analysis procedures are crucial
Examples of category 2 problems

• Metal forming, crash and crush analyses in the automobile industries

• These types of problems can now be solved much more reliably and efficiently than just a few years ago
Reduce Calculation time

- Use symmetry, anti-symmetry, beams, frames, shells, and trusses
- Compare FEA results with mechanics of materials theory
- Analytic approximation
- Then use them to dependently validate a more complex study

Needle insertion for drug delivery inside a tissue
Fluid-structural analyses
http://uhatmanoa.wixsite.com/ammi/copy-of-smp
Modeling and Optimization of Airbag Helmets for Preventing Head Injuries in Bicycling

Bicycling is the leading cause of sports-related traumatic brain injury (TBI)

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M. Kurt, K. Laksari, C. Kuo, G. Grant, D. Camarillo, "Modeling and Optimization of Airbag Helmets for Preventing Head Injuries in Bicycling", Annals of Biomedical Engineering (2016) [Link].
Example: Analysis of a Z-section Beam

Z shape cantilever beam
length of \( L = 500 \) mm.
thickness of \( t = 5 \) mm,
each flange has a length of \( a = 20 \) mm,
depth of \( h = 2a = 40 \) mm.
loaded by a vertical force \( P = 500 \) N
Bending moment, about the x-axis \( M = P (L - z) \) where \( z \) is the distance from the support.

\( \sigma_z \)

For symmetric sections: stress is zero at the neutral axis.

The load \( P \) causes a moment and a shear force.

Maximum tension along the top edge, and a compression along the bottom.

- Transverse shear stress (\( \tau \)) varies parabolically through the depth and has its maximum at the neutral axis.
1D Cantilever Beam Theory

\[ \sigma_z = \frac{My}{Ix} \]

\[ \tau = \frac{PQ}{tIx} \]

\( I_x \) is the second moment of inertia
\( Q \) is the first moment of the section at a distance, \( y \), from the neutral axis.
For this section \( I_x = (2t a^3) / 3 \)

The maximum tension will occur at \( y = a + t/2 \), while compression occurs at \( -y \)

\( U_y = \frac{PL^3}{3EI_x} = 7.8\text{mm} \)
3D Beam Theory

The more general non-symmetric beam 1D displacement predictions are

\[ U_y = \frac{PL_y(L - z)^3}{6EI_x l_y - l_{xy}^2} + c_0z + c_1 = \frac{P[(L - z)^3 + 3zL^2 - L^3]}{7Ea^3} \]

\[ U_x = \frac{PIL_{xy}(L - z)^3}{6E(I_x l_y - l_{xy}^2)} + c_2z + c_3 = \frac{-3U_y}{2} \]

\[ U_{y_{\text{max}}} = \frac{-2PL^3}{7Ea^3}, U_{x_{\text{max}}} = \frac{-3U_{y_{\text{max}}}}{2}, U_{\text{max}} = \frac{0.515PL^3}{Ea^3} \]

\[ U_y = -0.0045, U_x = 0.0067, \text{ and } U_{\text{max}} = 0.0081 \text{ meters} \]

\[ \sigma_z = 17.9e7 \text{ MPa} \]